

Cross Product and Triple Product

Algebraic definition of the cross product. If $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and $\vec{w} = \langle w_1, w_2, w_3 \rangle$, then we define $\vec{v} \times \vec{w}$ to be $\langle v_2w_3 - v_3w_2, v_3w_1 - v_1w_3, v_1w_2 - v_2w_1 \rangle$.

There is a handy way of remembering this definition: the cross product $\vec{v} \times \vec{w}$ is equal to the determinant

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = \begin{vmatrix} v_2 & v_3 \\ w_2 & w_3 \end{vmatrix} \vec{i} - \begin{vmatrix} v_1 & v_3 \\ w_1 & w_3 \end{vmatrix} \vec{j} + \begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix} \vec{k}$$

Note: The cross product is only defined for three-dimensional vectors.

1. For this problem, let $\vec{v} = \langle 1, 2, 1 \rangle$ and $\vec{w} = \langle 0, -1, 3 \rangle$.

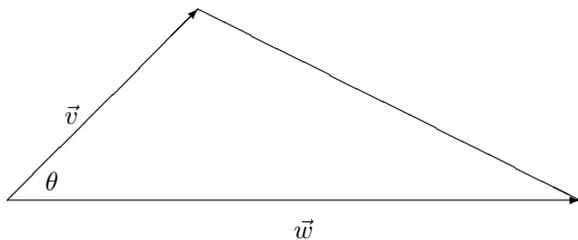
(a) Compute $\vec{v} \times \vec{w}$.

(b) Compute $\vec{w} \times \vec{v}$.

(c) Let $\vec{u} = \vec{v} \times \vec{w}$, the vector you found in (a). What is the angle between \vec{u} and \vec{v} ? \vec{u} and \vec{w} ?

2. In general, what is the relationship between $\vec{v} \times \vec{w}$ and $\vec{w} \times \vec{v}$?

3. Any two vectors \vec{v} and \vec{w} which are not parallel determine a triangle, as shown. What is the relationship between the area of the triangle and $\vec{v} \times \vec{w}$?



4. If \vec{v} and \vec{w} are parallel, what is $\vec{v} \times \vec{w}$?

5. If the scalar triple product $\vec{u} \cdot (\vec{v} \times \vec{w})$ is equal to 0, what can you say about the vectors \vec{u} , \vec{v} , and \vec{w} ?

6. Find an equation for the plane which passes through the points $(1, 0, 1)$, $(0, 2, 0)$, and $(2, 1, 0)$.

7. True or false: If $\vec{u} \times \vec{v} = \vec{u} \times \vec{w}$, then $\vec{v} = \vec{w}$.

8. True or false: If $\vec{v} \times \vec{w} = \vec{0}$ and $\vec{v} \cdot \vec{w} = 0$, then at least one of \vec{v} and \vec{w} must be $\vec{0}$.